Instantons and the endpoint of the lepton energy spectrum in charmless semileptonic B decays

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Abstract

A recent calculation by Chay and Rey has shown that instantons may make a significant contribution to the lepton energy spectrum near its endpoint. Using an ansatz borrowed from the study of high energy baryon number violating processes, we investigate whether these corrections could spoil the relation between the nonperturbative contributions to this spectrum and to the photon energy spectrum in radiative B decays. We find, in general, that this universality may well fail unless the spectrum is smeared over a region which is considerably larger than had previously been thought necessary. This result affects the possibility of performing a reliable measurement of V_{ub} using inclusive decays.

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I. INTRODUCTION

There has been considerable recent interest in the study of the endpoint spectra of charmless semileptonic and radiative B meson decays. This effort is motivated by the fact that in order to remove overwhelming backgrounds due to decays to charmed states, stringent cuts must be applied to the data, which restrict the experimental analysis to within a few hundred MeV of the kinematic endpoint. Hence it is crucial to understand in as much detail as possible the theoretical shape of the lepton or photon energy spectrum in the endpoint region, if one is to use these processes to extract reliably short-distance couplings such as Kobayashi-Maskawa matrix elements.

The current theoretical analysis makes use of the Operator Product Expansion (OPE) and the Heavy Quark Effective Theory (HQET) [1–3]. Within this context one may compute a variety of corrections to the simple free quark decay picture, both perturbative $\mathcal{O}(\alpha_s^n)$ and nonperturbative $\mathcal{O}(\Lambda^n/m_b^n)$ in origin. An important result of this analysis is that the leading nonperturbative power corrections in the endpoint region may be resummed into a universal shape function, which describes the distribution of the light-cone momentum of the b quark inside the B meson [4,5]. Since the same nonperturbative matrix elements describe the endpoints of both the lepton spectrum in charmless semileptonic decays and the photon spectrum in radiative decays, it is possible in principle to use a measurement of the photon spectrum to predict the shape of the lepton spectrum and thereby allow for a model-independent measurement of V_{ub} [4,6].

This relation is useful, of course, only if the dominant contribution to the shape of the lepton endpoint spectrum actually comes from the nonperturbative power corrections. One possible source of trouble is radiative corrections, which near the endpoint suppress the theoretical cross-section by a factor $\exp[-\frac{2\alpha_s}{3\pi}\ln^2(1-y)]$, where $y = 2E_\ell/m_b$ is the scaled lepton energy. For y sufficiently close to 1, this Sudakov suppression dominates the theoretical spectrum; whether this is true over the entire experimentally allowed window is less clear [6]. There has been recent progress toward resumming the leading and subleading Sudakov logarithms, which would reduce considerably the uncertainty due to this effect [7].

Another potential source of large corrections near $y = 1$ is instanton effects. Chay and Rey [8] have recently computed the one-instanton contribution to inclusive B decays, in the dilute gas approximation (DGA). Their conclusion was that for charmless semileptonic decays this contribution diverges severely at $y = 1$, while it is small and under control for radiative decays. Unfortunately, their suggestion that one regulate this divergence by considering the energy spectrum only in the region $y < 1 - \delta$, where $\delta \approx 0.16 \sim 0.20$, is not necessarily practical, given that the experimental analysis is restricted kinematically to the region $y \gtrsim 0.85$. In the region of experimental interest, the effect of instantons is potentially large and dangerous. Unfortunately, it is also the region in which the DGA begins to break down and multi-instanton processes become important.

In this paper we will investigate whether instantons spoil the relationship between the radiative and semileptonic endpoint spectra in a way that necessarily destroys its phenomenological usefulness. We will adopt an approach used in similar situations in the study of baryon number violation in high energy collisions [10], in which we use the one-instanton result as a guide to an ansatz for the multi-instanton contribution. This ansatz contains a small number of physical parameters, and we will investigate the size of instanton effects

as a function of these parameters. We will consider both the overall magnitude of the instanton contribution and the order-by-order behaviour of its moments, as compared to the nonperturbative corrections which arrive from higher order terms in the OPE.

The limitations of such an approach are clear. We will be dealing not with the true multi-instanton cross-section, which has not been computed, but with an ansatz which has been inspired by a one-instanton calculation which is valid in a different region. Nonetheless, we will come to conclusions which we believe are robust, and which indicate that large instanton corrections to the shape of the endpoint spectrum may be difficult to avoid.

II. THE ONE-INSTANTON CALCULATION

We begin by summarizing the calculation of Chay and Rey [8] of the effect of a single instanton on the lepton and photon energy spectrum. In the context of the OPE, the decay width is determined by the correlator of two quark bilinears. For example, for the process $B \to X_u \ell \nu$, the differential decay rate is given by

$$d\Gamma = \frac{G_F^2}{4m_b}|V_{ub}|^2 W^{\mu\nu} L_{\mu\nu} d(\text{P.S.}) , \tag{2.1}$$

where $L_{\mu\nu} d(\text{P.S.})$ is the product of the lepton matrix elements with a lepton phase space measure, and

$$W^{\mu\nu} = -2 \operatorname{Im} \left\{ i \int d^4 x e^{iq\cdot x} \langle B \,|\, T\{\bar{b}\gamma^\mu(1-\gamma^5)u(x), \bar{u}\gamma^\nu(1-\gamma^5)b(0)\} \,|\, B\rangle \right\} \tag{2.2}$$

describes the interactions of the quarks [1]. The correlator is developed in a simultaneous expansion in α_s and the off shell momentum of the u quark, which is of order m_b everywhere but at the boundaries of phase space. In terms of the scaled variables $y = 2p_b \cdot k_\ell/m_b^2$ ($= 2E_\ell/m_b$ in the B rest frame) and $\hat{s} = (k_\ell + k_\nu)^2/m_b^2$, these boundaries are at $y = \hat{s}$ and $y = 1$.

The calculation of the correlator (2.2) in the dilute instanton background gives the instanton contribution to the decay width [8]. The instanton contribution enters as a contribution to the coefficients of the operators which appear in the OPE. The computation involves an integration over the instanton size ρ, which diverges in the infrared. Chay and Rey deal with this divergence by expanding the integrand in $1/\rho$ and interpreting the finite number of divergent terms as contributions to the matrix elements of operators in the OPE. This is appropriate insofar as the divergent contribution of large instantons is presumably regulated physically by the infrared growth of the QCD self-coupling. The terms which are infrared convergent and hence calculable are interpreted as contributions to the coefficient functions in the OPE.

With some mild additional approximations, Chay and Rey derive an expression for the leading one-instanton contribution to the doubly differential decay width,

$$\frac{1}{\Gamma_0} \frac{d^2\Gamma_{\text{inst}}}{d\hat{s}dy} = A y^5 \frac{5\hat{s} - (1-y)(y-\hat{s})}{(1-y)^6(y-\hat{s})^5} , \tag{2.3}$$

where $\Gamma_0 = G_F^2|V_{ub}|^2 m_b^5/192\pi^3$. The constant A depends on the quark masses, the QCD scale Λ and the number of light flavors, and is estimated by Chay and Rey to be

$$A = \left(\frac{19.2\,\text{GeV}}{m_b}\right)^3 \left(\frac{\Lambda}{m_b}\right)^9 \approx 6.7 \times 10^{-8}\,, \tag{2.4}$$

for $\Lambda = 350\,\text{MeV}$ and $m_b = 4.5\,\text{GeV}$. Note that the expression (2.3) scales naïvely as m_b^{-12} and has strong divergences as $y \to 1$ and $y \to \hat{s}$. For the radiative decay $B \to X_s\gamma$, under the same approximations, Chay and Rey find the one-instanton contribution to the photon energy spectrum,

$$\frac{1}{\Gamma_{0,\gamma}} \frac{\mathrm{d}\Gamma_{\text{inst},\gamma}}{\mathrm{d}y_\gamma} = \left(\frac{26.1\,\text{GeV}}{m_b}\right)^3 \left(\frac{\Lambda}{m_b}\right)^9 \frac{y_\gamma^3}{(1-y_\gamma)^7}\,, \tag{2.5}$$

where $y_\gamma = 2p_b \cdot k_\gamma/m_b^2$ is the scaled photon energy in the B rest frame, and $\Gamma_{0,\gamma}$ is the lowest order free quark radiative decay width. Again, the instanton contribution diverges strongly as $y_\gamma \to 1$. However, only integrals of Eq. (2.5) are actually meaningful, as we discuss below.

The divergent behaviour of the instanton contribution at the edges of phase space has a straightforward origin. Once the infrared divergences have been subtracted, the contribution of instantons to the coefficient functions comes from small instantons of typical size $\rho \leq 1/|Q|$, where the scale Q is determined by the momentum of the final quark propagating in the instanton background. Since the instanton contribution contains the suppression factor $\exp(-2\pi/\alpha_s(Q^2))$, it is important only when Q^2/Λ^2 is of order one, that is, when the invariant mass of the hadronic final state is of order the QCD scale rather than the bottom mass. This occurs at the boundaries of phase space, where the final state light quark is driven to its mass shell. Hence this is the region where instanton effects become not only significant, but divergent.

An alternative approach to this calculation would be to cut off all instantons with $\rho \gtrsim 1/m_b$, which would suppress the one-instanton contribution by $(\Lambda/m_b)^9$, independent of y. Such a cutoff might be natural for a diagram in which all propagators are in the instanton background, such as the calculation of the polarization operator in e^+e^- annihilation. However, for semileptonic and rare B decays the instanton insertions are only in one propagator, and the choice of cutoff should be governed solely by the kinematics of this light quark, not by the total energy released in the decay. Effectively, we cut off the integral over ρ at $\rho \sim 1/|Q|$, leading to a differential rate which is unsuppressed at the lepton energy endpoint.

Essentially, the question is whether the integral over ρ should be performed before or after the integration over loop momenta. If one performs the loop integration first, then the only remaining *external* momentum scale is set by m_b, and a cut on $\rho \gtrsim 1/m_b$ would seem natural. If, however, we perform the integral over ρ first, then a loop momentum-dependent cutoff prescription such as we use becomes possible, and in this case, for the reasons discussed above, is preferred.

In the case of $B \to X_s\gamma$ decays, it is possible to regulate the divergence in Eq. (2.5) in such a way that the contribution of instantons to the *total* decay rate is finite and, in fact, negligibly small [8]. The situation is similar to that of the contributions of Sudakov double logarithms to this process [5]; so long as the cut $y > y_c$ on the photon momentum is not too stringent ($y_c \lesssim 0.85$ will do), it is possible to analytically continue the phase space integral away from any resonance region. Once this has been done, the one-instanton contribution is small, and calculable, everywhere. Hence, the one-instanton result gives a reliable estimate of instanton contributions to integrated quantities such as the total decay width.

Unfortunately, this procedure will not work for $B \to X_u \ell \nu$ decays. In this case one must perform an integration in the variable $p_b \cdot (k_\ell + k_\nu)/m_b^2$, and near the boundary of phase space the contour cannot be deformed away from the resonance region [1,8]. (This happens both because the endpoint of the integration is a function of y and $\hat{s}$ and cannot be adjusted by hand, and because in certain regions of phase space the contour is pinched between two cuts.) As a result, in this regime we do expect a large instanton contribution.

Of course, in this region the OPE itself breaks down, as corrections from operators of higher twist become important. However, the one-instanton correction has such severe divergences in this region that instantons become important even when $Q^2 > \Lambda^2$, so it is not unreasonable to expect that the OPE analysis gives the correct order of magnitude of instanton effects.

Even in the absence of nonuniversal higher twist contributions, instantons contribute differently to semileptonic and rare B decays. The reason is essentially the dependence, in the semileptonic case, of the boundary point of the contour of integration on the kinematics of the leptons. Such a dependence does not affect universality in the absence of instanton effects, because the OPE is an expansion in inverse powers of Q^2 and therefore has a pole behaviour, which is insensitve to the shape of the contour of integration. Instanton zero modes, by contrast, give $\log Q^2$ contributions to coefficient functions, so instanton corrections depend explicitly on the position of the boundary point of the contour.

III. THE MULTI-INSTANTON ANSATZ

The calculation discussed in the previous section shows that the one-instanton contribution to the semileptonic differential decay rate becomes of the same size as the lowest order result in the region $y \gtrsim 0.9$, and diverges in the limit $y \to 1$. In this region, then, we cannot trust any more the one-instanton result, and we must include multi-instanton corrections as well. Unfortunately, there is not at present a technology for performing such calculations.

The situation is similar to one obtaining in the study of high energy collisions, where the one-instanton correction to the total cross section grows exponentially with energy and violates unitarity in the multi-TeV region. Possible ways of treating this problem have been discussed widely in the literature (for a review, see Ref. [9]). Most likely, multi-instanton corrections stop this dangerous growth and unitarize the amplitude at high energies. While the detailed behaviour of the multi-instanton contribution is, of course, not known, one makes a hypothesis as to its qualitative form. It is assumed that the instanton-mediated cross section has a threshold behaviour: it is dominated by the one-instanton contribution below the threshold, and hence exponentially suppressed; it reaches the unitarity bound in the threshold region; and it stays almost constant above threshold, in the multi-instanton regime. These properties lead to an ansatz for the full instanton contribution, which is a step function with support above the threshold [10]. This ansatz has two parameters: the width of the step, corresponding to the position of the threshold, and the height, corresponding to the strength of the unitarized amplitude. While such an ansatz is obviously extremely crude, it incorporates the one useful piece of information which may be extracted from the one-instanton calculation: the energy at which the instanton contributions become large. Because of the rapid rise of the one-instanton contribution with energy, this threshold is actually predicted fairly reliably.
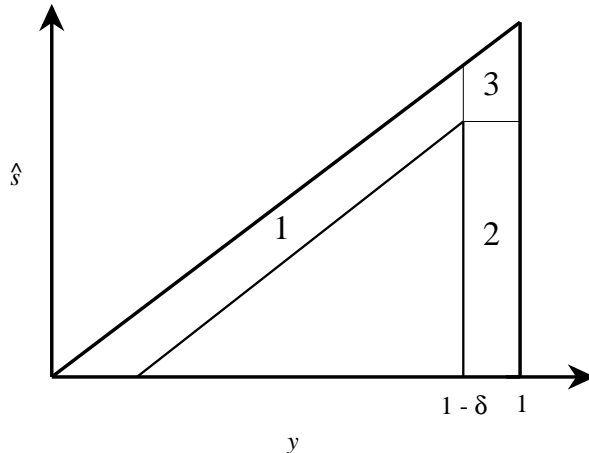
FIG. 1. The one-instanton calculation is reliable only within the inner triangle. In regions 1, 2 and 3, we replace the one-instanton result with the multi-instanton ansatz, as explained in the text.

We will follow an analogous procedure in our discussion of the instanton correction to the semileptonic spectrum. We will arrive at an ansatz for the multi-instanton contribution which is equally crude, but which we hope will again incorporate correctly the information provided by the one-instanton calculation. To try to account as honestly as possible for the large and uncontrolled uncertainty in our ansatz, we will vary the parameters which define it over ranges which, in our opinion, are quite generous.

We note briefly the claim in the literature [11] that the one-instanton result actually unitarizes "prematurely", with its growth stabilized when it is still exponentially small. This proposal is still somewhat controversial [9,12]; if correct, it will result in a strong suppression of all instanton effects. We will not address this issue further, except to note that in our ansatz we allow for a significant variation in the overall normalization. This normalization can be taken in principle to include the effect of premature unitarization, although if the phenomenon is real then it may lead to a stronger suppression than we consider below.

We begin with the expression (2.3) for the one-instanton contribution to the doubly differential decay rate. This expression is severely divergent as $\hat{s} \to y$ and $y \to 1$, and particularly so when both limits are taken simultaneously. Let us suppose, then, that we believe the one-instanton calculation only over the region $R = \{0 \le \hat{s} \le y - \delta,\ \delta \le y \le 1 - \delta\}$, where $\delta \ll 1$. Outside of R, multi-instanton configurations probably regularize the otherwise divergent decay rate, and the largest instanton contributions to the rate actually come from this region at the boundary of phase space. Hence we replace the one-instanton result by a step function ansatz, as shown in Fig. 1. In region 1 of Fig. 1, we take as our ansatz Eq. (2.3), with $\hat{s} = y - \delta$; in region 2, we take Eq. (2.3) with $y = 1 - \delta$; and in region 3, we take Eq. (2.3) with $y = 1 - \delta$ and $\hat{s} = 1 - 2\delta$. Elsewhere, the instanton-mediated decay rate is dominated by the small one-instanton contribution, and is taken to vanish.

Next, we integrate this ansatz over $\hat{s}$ to get a preliminary ansatz for $d\Gamma_{\text{inst}}/dy$. To within

6

a factor of two* for $\delta \lesssim 0.3$, we find the simple result

$$\frac{1}{\Gamma_0}\frac{d\Gamma_{\text{inst}}}{dy} \approx \begin{cases} 5Ay^5/(1-y)^6\delta^4 & \text{if } y < 1 - \delta \\ 5A(1-\delta)^5/\delta^{10} & \text{if } 1 - \delta \leq y < 1 \end{cases} \tag{3.1}$$

Since the function falls steeply for $y < 1 - \delta$, we make the further simplification of setting the function to zero in this region. The final form for our ansatz, then, is a step function which takes the form

$$\frac{1}{\Gamma_0}\frac{d\Gamma_{\text{inst}}}{dy} = \frac{\nu}{\delta}\{\Theta(1-y) - \Theta(1-y-\delta)\}. \tag{3.2}$$

The height $\rho = \nu/\delta$ of the step is given by the second case of Eq. (3.1).

   We stress that this ansatz for the multi-instanton contribution to the width is extremely crude. However, it does contain certain useful information gleaned from the one-instanton result. Because of the extremely strong dependence of ρ on δ, we actually find a reasonable constraint on the width. We assume that multi-instanton contributions come in and regularize the width when ρ gets larger than some value ρ_0, and let us take the very generous range $\frac{1}{10} < \rho_0 < 10$. Then we find, from the expression (3.1), that $0.16 < \delta < 0.24$. This is the range of δ which we will allow in Eq. (3.2).

   It is somewhat more difficult to set a reasonable range for the area ν. We would like to allow for the significant uncertainty in the derivation of Eq. (3.1), without losing entirely the strong dependence of $\nu = \rho\delta$ on δ, which is physical. Our prescription will be to introduce an *ad hoc* multiplicative factor in the normalization of the ansatz, and to consider a wide variation in its value. Hence we will consider the functions

$$\nu_i = \rho_i\delta = c_i \cdot 5A\frac{(1-\delta)^5}{\delta^9}, \tag{3.3}$$

where $c_1 = 1$, $c_2 = \frac{1}{10}$, and $c_3 = \frac{1}{100}$. These three functions cover a variation of two orders of magnitude in the true size of the multi-instanton contribution, compared with our naïve ansatz (3.2). Combined with the restriction $0.16 < \delta < 0.24$ obtained above, we find $0.0002 < \nu < 1.6$. While the upper limit is not to be taken seriously, we believe that $\nu \gtrsim 0.0002$ represents a reasonable lower limit on the size of multi-instanton effects.

IV. IMPLICATIONS FOR UNIVERSALITY

   Given this ansatz for the multi-instanton contribution, what are the implications for the measurement of V_{ub}? When instanton effects are included, do they dominate the shape of the endpoint spectrum, or is this shape still determined by the nonperturbative power corrections? In order to frame this question properly, we must consider both the overall size of the instanton contribution and its effect on the moments of the spectrum.

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  *We will soon vary the rate over a range much larger than this, so the approximations involved in obtaining this simple and convenient expression are relatively harmless.

The leading power corrections to the lepton energy spectrum may be expanded in a series of singular functions near $y = 1$,

$$\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma_{\mathrm{th}}}{\mathrm{d}y} = B_0\theta(1-y) + B_1\delta(1-y) + B_2\delta'(1-y) + \ldots, \qquad (4.1)$$

where $B_n \sim (\Lambda/m_b)^n$. The singular parts of this expression may be resummed into a "shape function" $S(y)$ of width Λ/m_b, so Eq. (4.1) takes the form [4]

$$\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma_{\mathrm{th}}}{\mathrm{d}y} = 2y[F(y)\theta(1-y) + F(1)S(y)], \qquad (4.2)$$

where $F(y) = y(3-2y) + \mathcal{O}(\alpha_s)$ is a smooth function of y. It is convenient to define moments of the shape function,

$$M_n^{\mathrm{th}} = \int \mathrm{d}y\, S(y)(y-1)^n, \qquad (4.3)$$

for $n \geq 1$. The moments scale inversely with the bottom mass, $M_n^{\mathrm{th}} \sim (\Lambda/m_b)^n$. If the spectrum is smeared with a weighting function of width σ near $y = 1$, for example $w(y) \propto \exp[-(y-1)^2/2\sigma^2]$, then the result may be written as a sum of these moments,

$$\int \frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma_{\mathrm{th}}}{\mathrm{d}y}\, w(y)\mathrm{d}y = \int 2y^2(3-2y)w(y)\mathrm{d}y + 2\sum_{n=1}^{\infty}\frac{M_n^{\mathrm{th}}}{n!}w^{(n)}(1). \qquad (4.4)$$

It is this sum which is universal, in the sense that it appears both in the expression for the semileptonic endpoint spectrum and that for the photon energy spectrum in $B \to X_s\gamma$ transitions. If the smearing region σ is of order Λ/m_b, then all terms in the sum (4.4) are of the same order in the $1/m_b$ expansion [4–6].

Instanton corrections appear as an additional term in the shape function,

$$S_{\mathrm{inst}}(y) = \frac{1}{2\Gamma_0}\frac{\mathrm{d}\Gamma_{\mathrm{inst}}}{\mathrm{d}y}. \qquad (4.5)$$

The new contribution to the right hand side of Eq. (4.4) takes the form

$$\int \frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma_{\mathrm{inst}}}{\mathrm{d}y}\, w(y)\mathrm{d}y = 2\sum_{n=0}^{\infty}\frac{M_n^{\mathrm{inst}}}{n!}w^{(n)}(1), \qquad (4.6)$$

where

$$M_n^{\mathrm{inst}} = \int \frac{1}{2\Gamma_0}\frac{\mathrm{d}\Gamma_{\mathrm{inst}}}{\mathrm{d}y}(y-1)^n\mathrm{d}y = \int S_{\mathrm{inst}}(y)(y-1)^n\mathrm{d}y, \qquad (4.7)$$

in analogy with Eq. (4.3). Universality will continue to hold if this new nonuniversal term is subleading, in some sense, compared to the universal series generated by the power corrections. In fact, there are two criteria which we must impose. The simplest is the condition that the total instanton contribution be small compared to the parton model rate. How small this ought to be is somewhat a matter of taste. As an illustration, let us take $w(y)$ to be a Gaussian weighting function of width σ. Then the parton model rate is given by

$$A_0(\sigma) = \int 2y^2(3 - 2y)w(y)\mathrm{d}y \,, \tag{4.8}$$

while the contribution of the instantons depends on the ansatz parameter δ,

$$A_{\mathrm{inst}}(\sigma, \delta) = \int \frac{1}{\Gamma_0} \frac{\mathrm{d}\Gamma_{\mathrm{inst}}}{\mathrm{d}y} \, w(y)\mathrm{d}y \,. \tag{4.9}$$

If we require the inequality

$$A_{\mathrm{inst}}(\sigma, \delta) < \kappa A_0(\sigma) \,, \tag{4.10}$$

for some κ, then we obtain lower limits on the smearing region σ as a function of δ. These limits depend also on the normalization c_i of the multi-instanton ansatz. In Fig. 2 we show $\sigma_{min}(\delta)$ for $c_1 = 1$, $c_2 = \frac{1}{10}$, and $c_3 = \frac{1}{100}$, and for $\kappa = \frac{1}{5}$ (a somewhat strict condition) and $\kappa = 1$ (a much looser one).

The second criterion is somewhat more subtle, and concerns the behaviour of the moments at large values of n. That the moments M_n^{th} scale as $(\Lambda/m_b)^n$ follows immediately from the form of the operator product expansion [4]. The instanton contribution, by contrast, is not an expansion in the inverse power of a momentum, and there is no reason for the instanton moments M_n^{inst} to show such a behaviour. Indeed, since the one-instanton result grows so steeply as $y \to 1$, its moments at large n may be large compared to the parton model result, even if they are suppressed at small n. The same is true of our multi-instanton ansatz; a simple calculation yields

$$\left| M_n^{\mathrm{inst}} \right| = \frac{1}{2(n + 1)} \, \delta^n \nu(\delta) \,. \tag{4.11}$$

The instanton moments are only subleading compared to the power corrections at large n if $\delta < \Lambda/m_b$.

Since the natural width of the theoretical shape function $S(y)$ is Λ/m_b, this is just the condition that the instantons are concentrated in a region closer to the endpoint than the smearing given by the initial b quark motion in the B meson. If so, then we can neglect not only the total instanton contribution but also the contribution of the instantons to the shape of the endpoint spectrum. The estimate of δ which we obtained previously, $0.16 < \delta < 0.24$, does not always satisfy this condition, except for quite large values of the QCD scale, $\Lambda \sim 1\,\mathrm{GeV}$.

We can resolve this problem only by adjusting the width σ of the smearing function $w(y)$. Let us suppose that the dependence of σ on the bottom mass is given by

$$\sigma \sim \left(\frac{\Lambda}{m_b} \right)^{1-\epsilon} \,, \tag{4.12}$$

where $\epsilon \geq 0$. As discussed in Refs. [4–6], if $\epsilon = 0$ then all of the terms in the series (4.4) are of the same order in Λ/m_b, since $M_n^{\mathrm{th}} \sim (\Lambda/m_b)^n$ and $w^{(n)} \sim 1/\sigma^n$. For $\epsilon > 0$, all terms with $n \gtrsim 1/\epsilon$ are suppressed by at least Λ/m_b and may be neglected, since they are of the same order as other terms which were dropped earlier. Hence $\epsilon = 0$ provides a lower, but not an upper, limit on the size of the smearing region, as a function of m_b.

Now suppose that the condition $\delta < \Lambda/m_b$ is not satisfied. Then the higher moments M_n^{th} are power-suppressed compared to M_n^{inst}, and there is in general an integer n_{crit} such that for $n > n_{\text{crit}}$ we have $M_n^{\text{inst}} > M_n^{\text{th}}$. If we insist on a smearing region size corresponding to $\epsilon = 0$, for which moments at all n contribute equally to the sum (4.4), then this situation will obviously lead to trouble. Since for large n the sum of moments is dominated by the instanton terms rather than those from the parton level, there will be no useful relation between the smeared spectra in radiative and rare semileptonic decays. Certainly, it will still be possible to use the observed photon spectrum in $B \to X_s \gamma$ to predict certain contributions to the lepton energy spectrum in $B \to X_u \ell \nu$, but these contributions will not be the dominant ones. Instead, the uncertainty in the shape of the lepton energy spectrum will be dominated by the uncertainty in the contributions of multi-instanton processes, which, as we have seen, is very large indeed.

Instead, we must smear over a larger region about $y = 1$, corresponding to an exponent $\epsilon > 0$ in Eq. (4.12). If we do so, then all moments with $n \gtrsim 1/\epsilon$ are subleading and may be ignored. Hence if we choose $\epsilon > \epsilon_{\text{crit}} = 1/n_{\text{crit}}$, then by the time $M_n^{\text{inst}} = M_n^{\text{th}}$, both M_n^{inst} and M_n^{th} may be neglected. In order to suppress the high moments of the instanton contribution, then, we are led to require a smearing width σ which might be significantly larger than previously expected. The critical smearing depends on the size of the multi-instanton ansatz, and is parameterized by a function $\epsilon_{\text{crit}}(\delta)$.

We can estimate $\epsilon_{\text{crit}}(\delta)$ by comparing our multi-instanton ansatz to the result which is obtained in the ACCMM model [13]. The moments M_n^{th} in this model have been calculated by Neubert [4], with the result

$$M_n^{\text{th}} = \frac{n!!}{2^{\frac{n+1}{2}}} \left(\frac{p_F}{m_b} \right)^{n+1} , \qquad (4.13)$$

for n even. A best fit to the spectrum yields $p_F \approx 230 \, \text{MeV}$, and we take $m_b = 4.8 \, \text{GeV}$. (Because of the symmetries of the model, M_n^{th} vanishes for n odd. The model also has the curious feature that the moments exhibit an $n!!$ growth, which alters somewhat the condition on δ. However, we remind the reader that the model is being used only as a somewhat crude comparison to an equally crude ansatz.) We obtain a value of n_{crit} by comparing the moments in Eqs. (4.11) and (4.13), and from this the critical smearing exponent $\epsilon_{\text{crit}}(\delta)$. The result in the ACCMM model, however, is that even under the worst assumptions, $\epsilon_{\text{crit}} \lesssim \frac{1}{10}$, so this effect turns out to be relatively unimportant. In fact, this is not so surprising: in our multi-instanton ansatz, we explicitly have cut off the strong divergence of the instanton contribution near the endpoint, so the effect of instantons on the shape of the endpoint spectrum is less important than their total contribution to the decay rate.

V. IMPLICATIONS AND DISCUSSION

Do the limits $\sigma_{min}(\delta)$, summarized in Fig. 2, indicate that instantons constitute an important effect on the endpoint spectrum? The answer to such a question depends crucially on precisely how it is posed. It is clear that since we are working with an extremely crude ansatz for the multi-instanton contribution, no numerical prediction which results is to be believed. However, the real hope of this analysis was to show that instanton effects are sufficiently *negligible* that the proposal to measure the shape of the photon energy spectrum
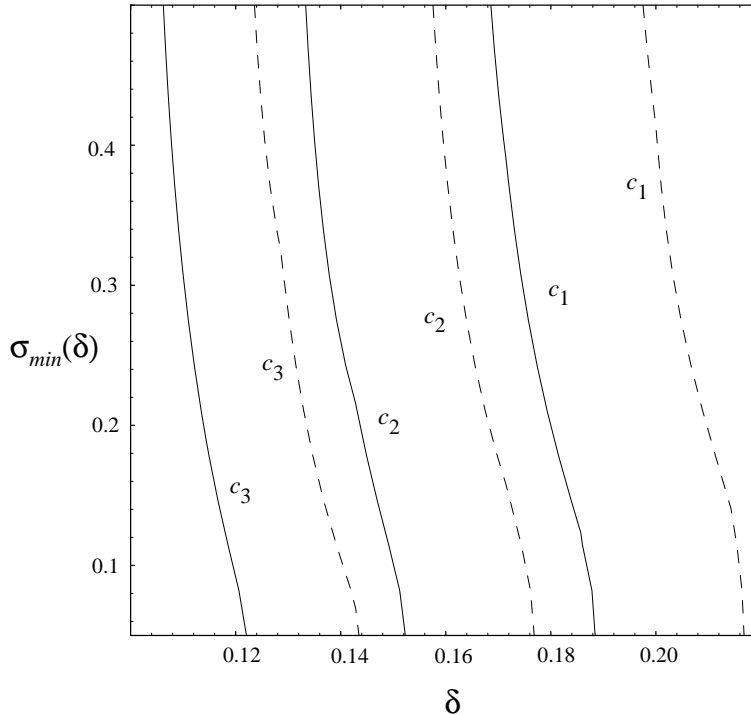
FIG. 2. The minimum smearing width $\sigma_{min}(\delta)$ for the multi-instanton ansatz. We show curves for the normalizations $c_1 = 1$, $c_2 = \frac{1}{10}$ and $c_3 = \frac{1}{100}$ of the multi-instanton contribution, and for $\kappa = \frac{1}{5}$ (solid curves) and $\kappa = 1$ (dashed curves).

in radiative B decays and use it to predict the shape of the endpoint spectrum in charmless semileptonic B decays is left unaffected.

As we see from Fig. 2, this is certainly not the case. For example, given the experimental constraints, the smearing region must satisfy $\sigma \lesssim 0.2$. If we now focus on the naïve normalization of the ansatz, $c_1 = 1$, and on the "loose" criterion $\kappa = 1$, we see that only for $\delta \gtrsim 0.18$ is $\sigma_{min} < 0.2$. For $\delta \lesssim 0.18$, the instanton effect dominates the nonperturbative shape function over the experimental smearing region. But recall from Section III that the one-instanton result indicates that the entire region $0.16 < \delta < 0.24$ is likely to be allowed. Note that our ansatz hardly has to be pushed to its extremes for the relationship between the shapes of the radiative and charmless semileptonic spectra to be spoiled. If we apply the "strict criterion" $\kappa = \frac{1}{5}$, then only the curve with the suppression factor $c_3 = \frac{1}{100}$ is acceptable over the entire range $0.16 < \delta < 0.24$. We conclude, then, that *there is no reason to believe that it is safe to neglect instantons in the analysis of the lepton energy endpoint spectrum.* Hence, we would have no particular confidence in the result of the proposed program to measure V_{ub} by comparing endpoint spectra, were it ever to be performed.

We stress that our result is interesting and important only in the *negative* sense. By no means do we claim to have calculated the effect of multi-instanton configurations, or even to have estimated them with any particular accuracy. What we have done, instead, is to analyze

11

a well-motivated multi-instanton ansatz honestly and conservatively. Within this ansatz, we have not found instanton effects uniformly to be negligible, from which we have concluded that neither are they *necessarily* negligible in the real world. We note that we would have reached a different conclusion, had considerable variation in the ansatz parameters yielded consistently negligible results.

Of course, one may take the point of view that instanton effects could as easily be tiny as large, and that in the absence of better evidence one should proceed on this more hopeful assumption. However, this naïve approach, if applied to the extraction of V_{ub}, would lead to a systematic uncertainty which is unknown and probably unknowable. On the other hand, perhaps our ansatz can be refined, or improved, or replaced by something closer to the truth. Eventually, perhaps, effects such as we have considered here may even be calculable. We certainly hope that such advances will one day prove instantons to be unimportant to the semileptonic endpoint spectrum, and the proposed experimental analysis to be unaffected. But unless good reasons arise for such confidence to be restored, we will consider the measurement of V_{ub} by the detailed analysis of the lepton energy endpoint spectrum to be intrinsically uncertain and untrustworthy.

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